Programming Math Software & Its Applications in Education

—Developments of KeTpic,KeTCindy,KetCindyJS—

Setsuo Takato

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Today's Contents

KeT=Kisarazu Educational Takato

and his happy friends

(1) Introduction

(2) Development of KETpic

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(4) Development of KETCindyJS

(5) Wasan Problems and KETCindy

(6) Conclusion

Introduction

Use of Mathematical Software

(1) Computer Algebra

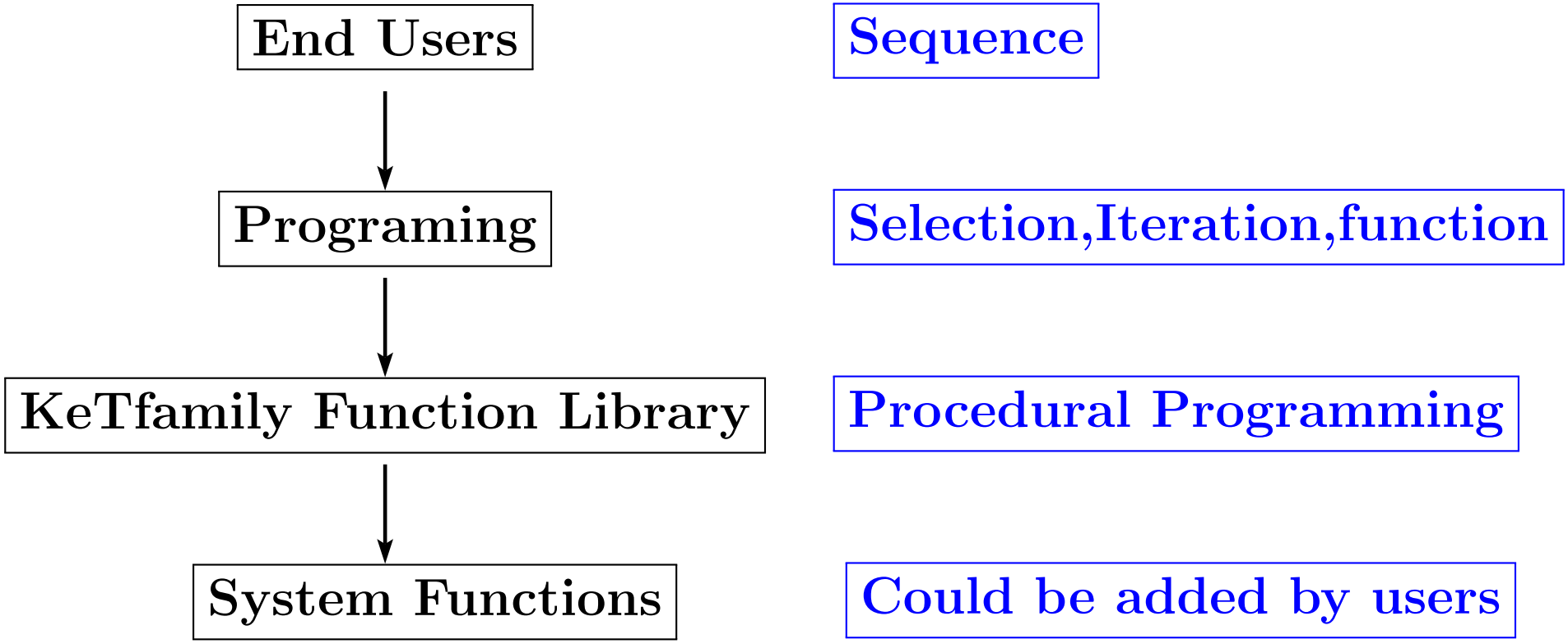
- It calculates the formula as is
- Maple,Mathematica,Maxima,Risa/Asir,...
- I initially used Maple in seminar-style classes

(2) Other Mathematical Software

- Scilab,R,Cinderella

(3) All of these support structured programming and list processing

Structured Programming



Developing Programming Skills

Until the 1980s I used BASIC

- Not structured, so limited to 1000 lines

Since the early 1990s I used Turbo Pascal5.0

- Was structured and allowed the creation of functions

In the late 1990s I bought Maple

- Tried to solve Wasan problems in my class
- Many case of a triangle could not be solved
- So I developed the MNR method
- It helped me improve my programming skills

Development and Use of KETpic

Use of TEXand Maple

(1) I continued to participate in the editing of the College of Technology textbook series

(2) TEXhas been used since the 2003 series

(3) We initially created the figures using WinTpic.

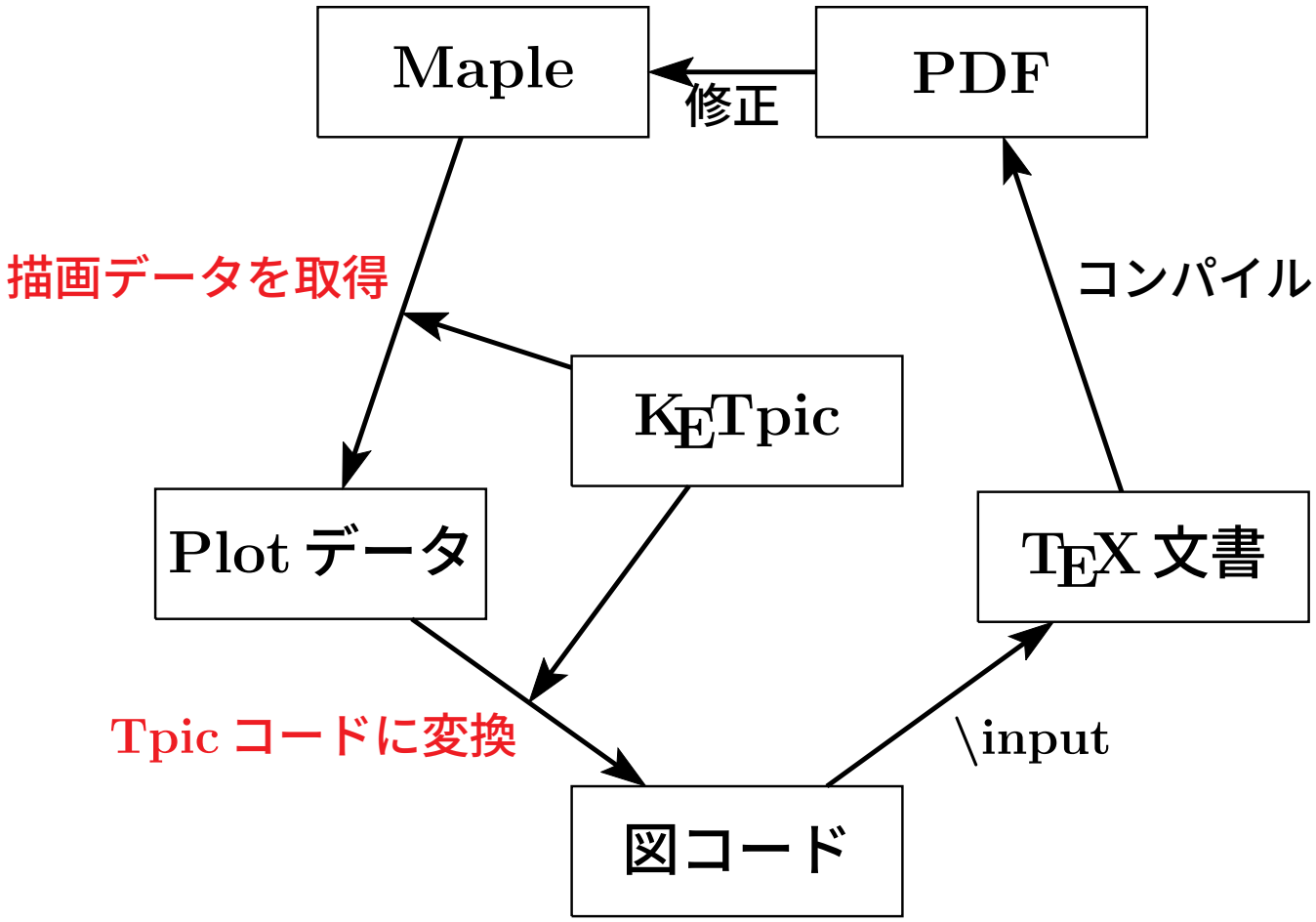
(4) Not able to draw spatial figures accurately

(5) I decided to use Maple to write out the tpic code.

- KETpic gets plot data for Maple
- It converts the data to tpic and output them

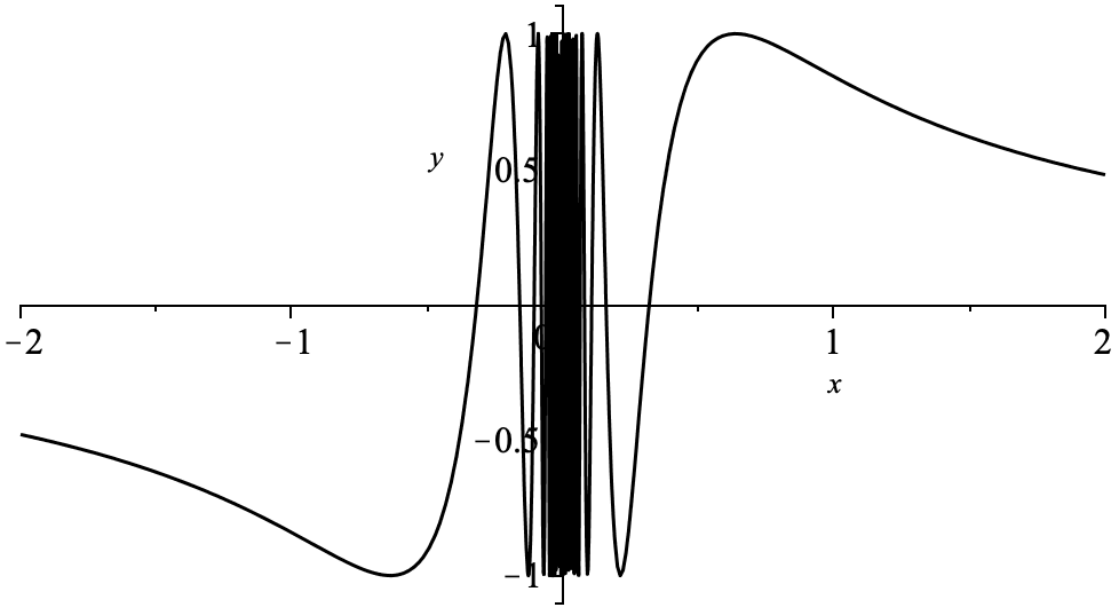
(6) We presented it at ICMS2006

KETpicDrawing Process



Example of Creating Codes

```
with(plots):
read cat(folder,‘ketpic.m‘):
setwindow(-2..2,-1.1..1.1):
g1:=plotdata(sin(1/x),
   x=XMIN..XMAX,numpoints=200):
windisp(g1):
```



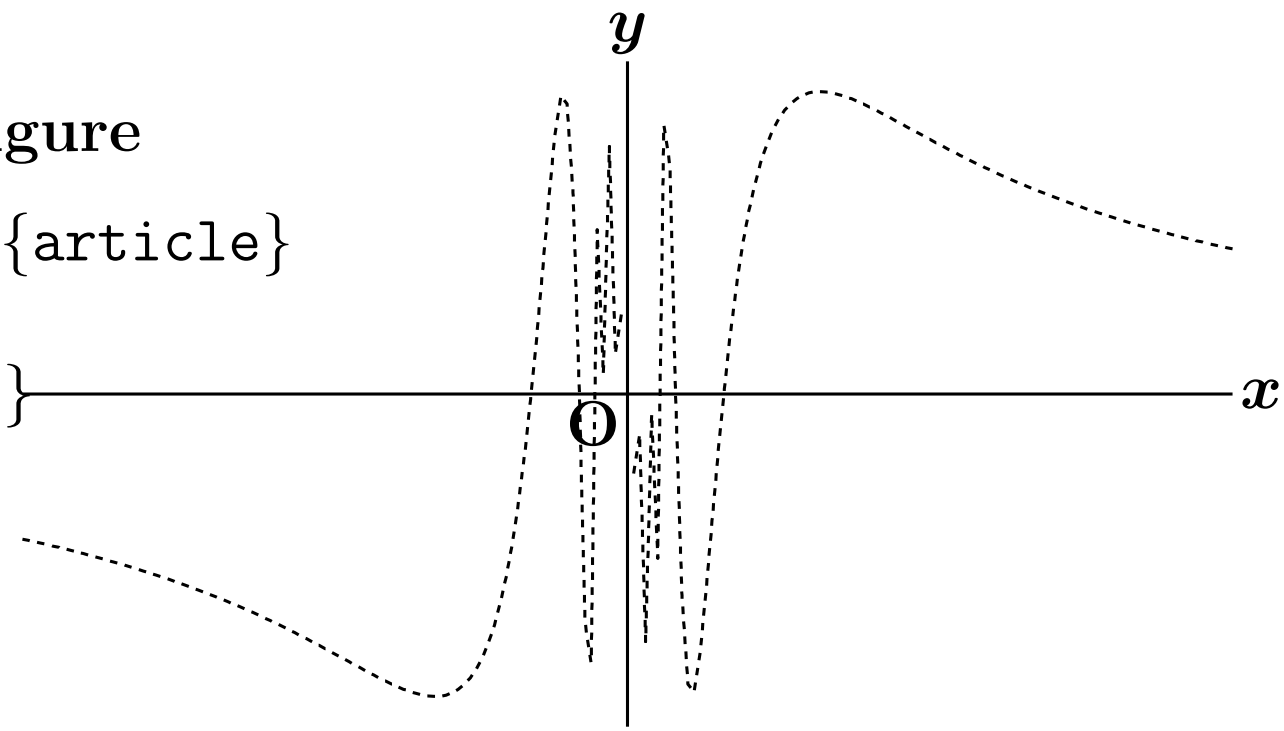
Example of Creating Figures

Continued with Maple

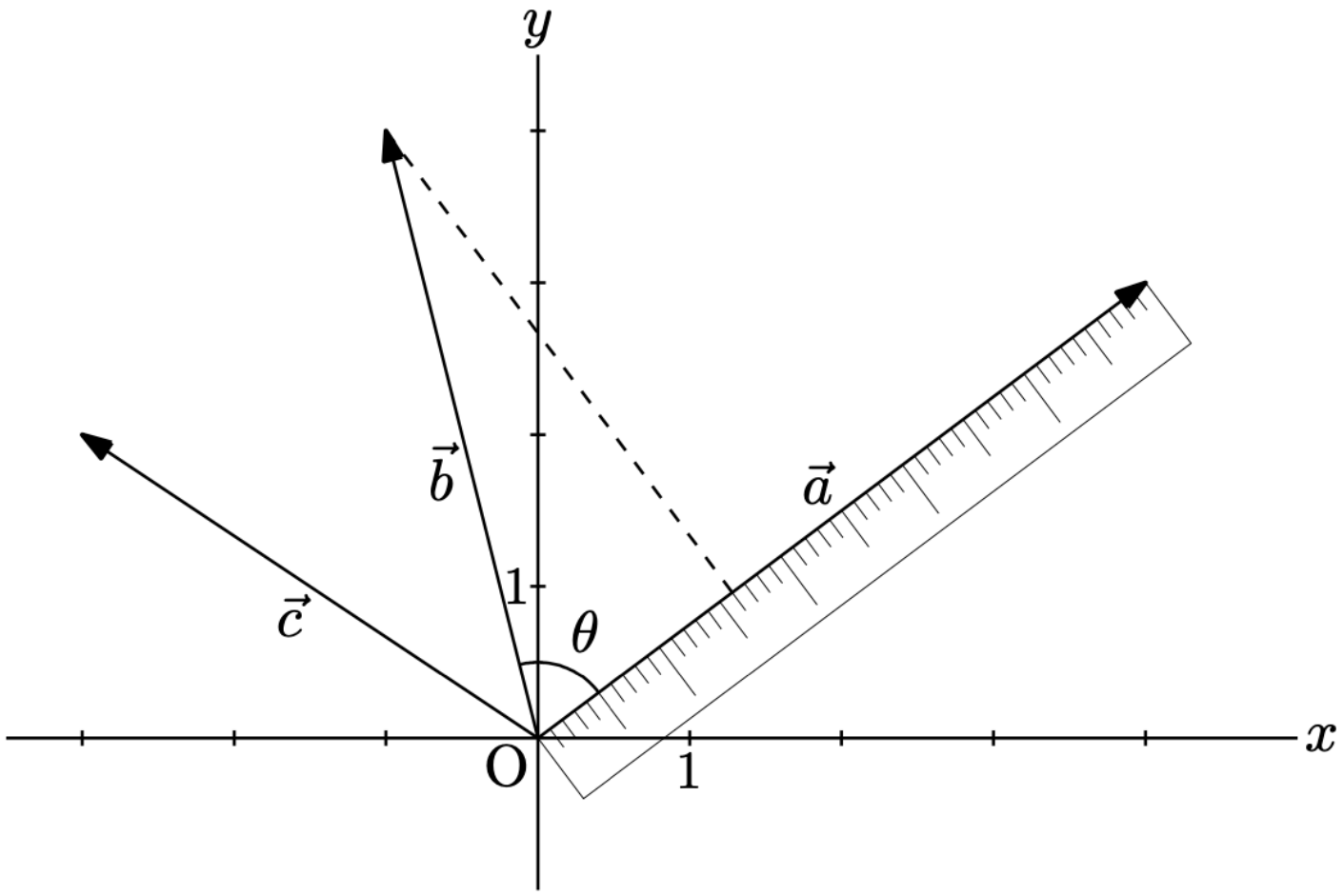
```
beginpicture("1cm"):
dashline(g1,0.5,0.5):
endpicture():
closepicture():
```

TEX Code and the Figure

```
\documentclass[a4]{article}
\newlength{\width}
\newlength{\Height}
\newlength{\Depth}
\begin{document}
\input{f1.tex}
\end{document}
```



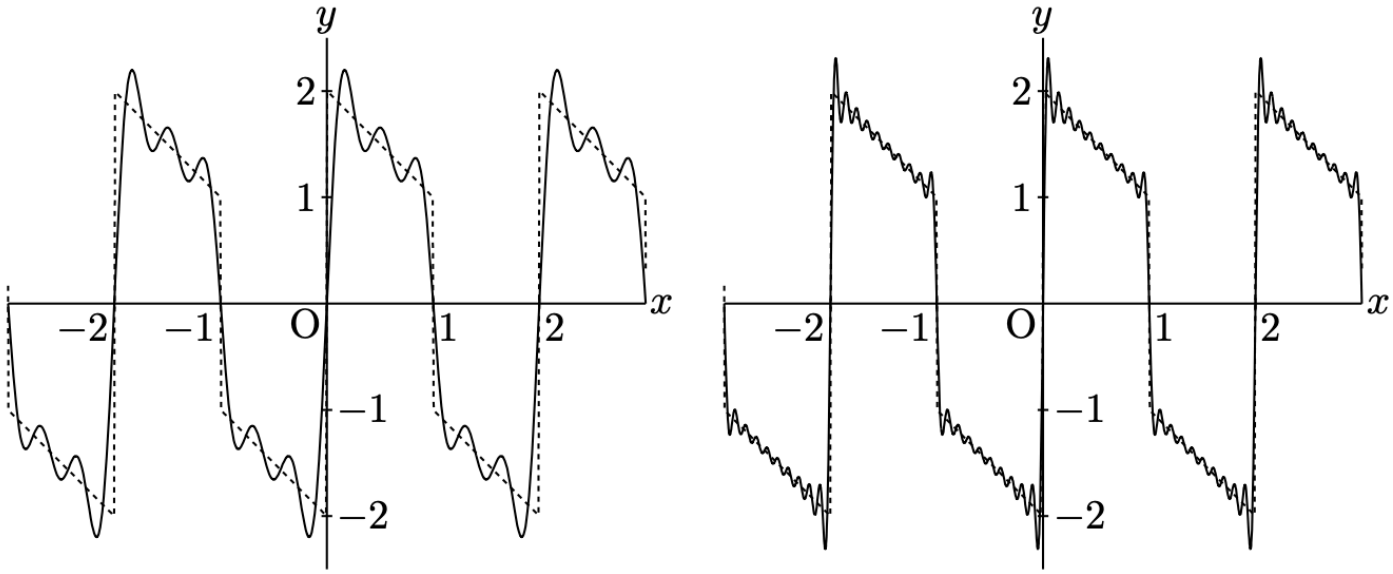
Example(1) Dot Product



$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = |\vec{a}| \times \text{Projection of } \vec{b}$$

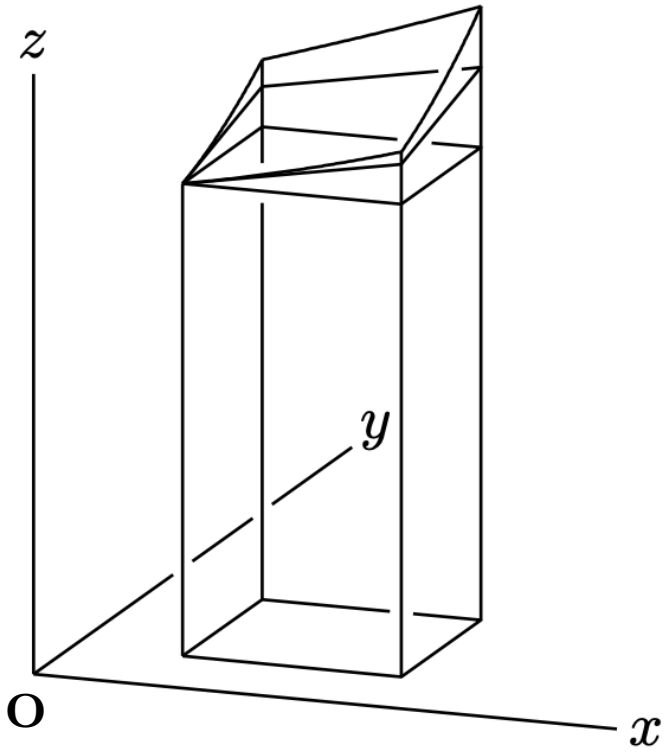
Example(2) Fourier Series

$$f_N(x) = \sum_{n=1}^{N} \frac{2(2-(-1)^n)}{n\pi} \sin n\pi x$$

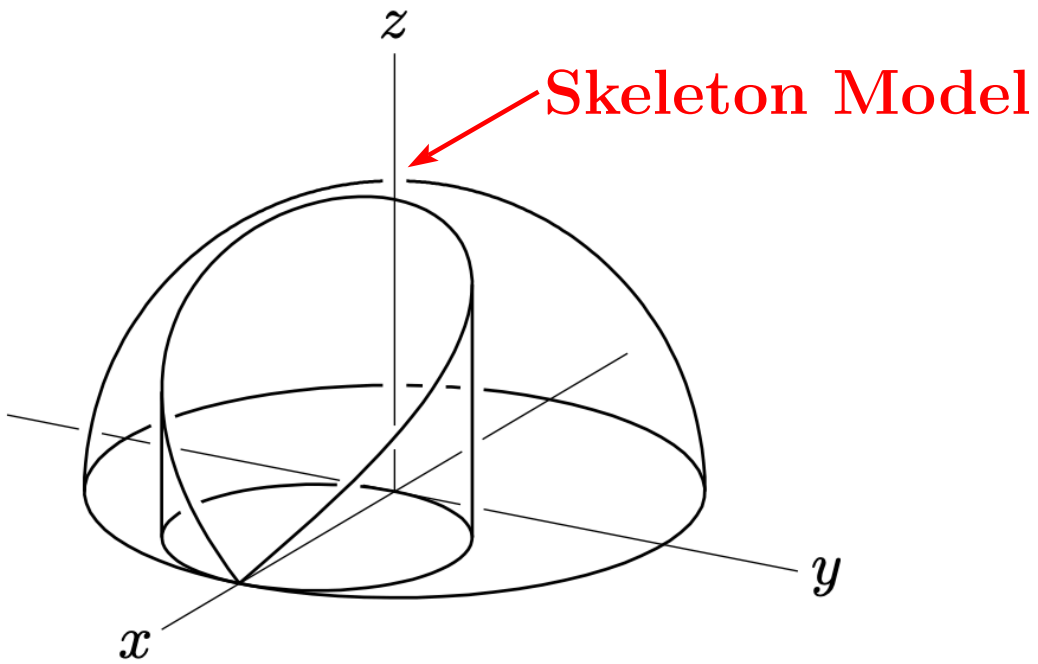


Gibbs Phenomenon

Example(3) Spacial Figures

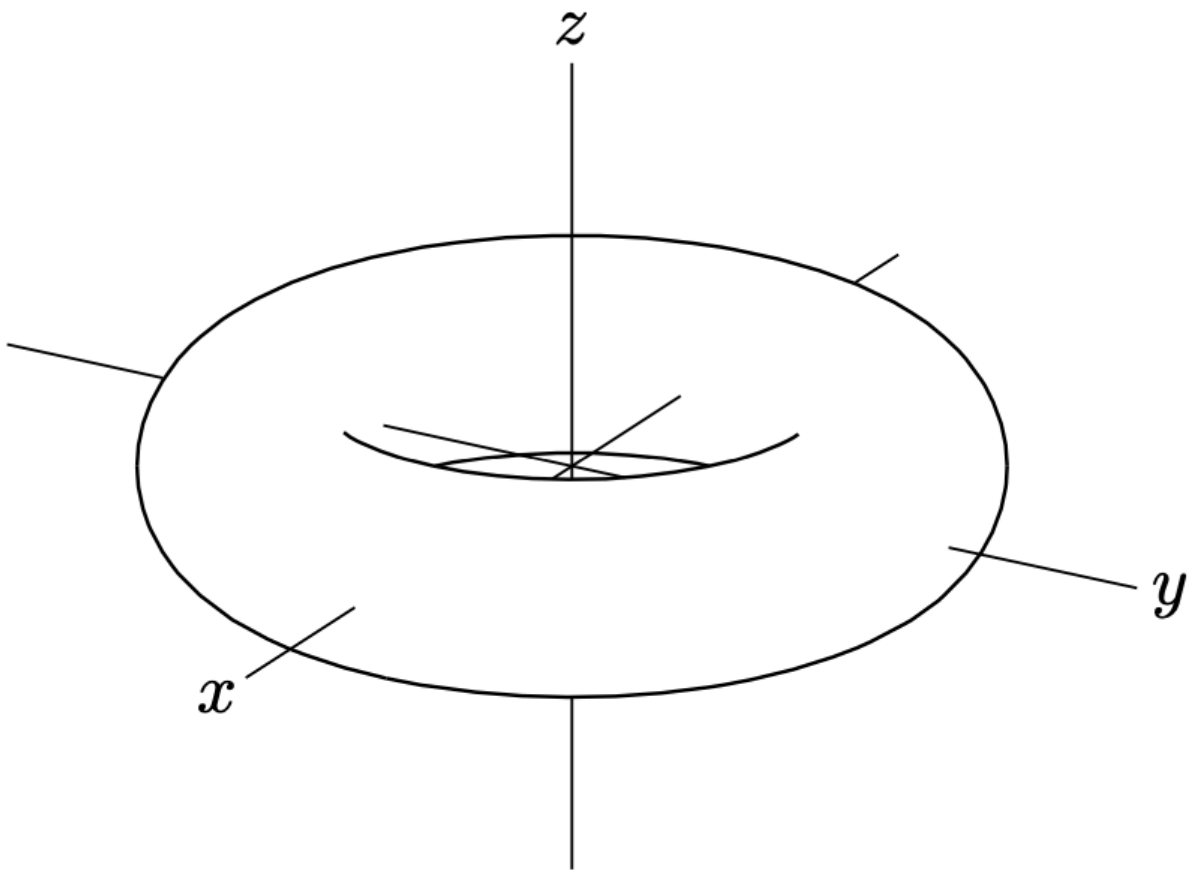


Meaning of Total Differential



Viviani Curve

Example(3) Surfaces(Torus)



Adding Axes

Expanding Functionalities of KETpic

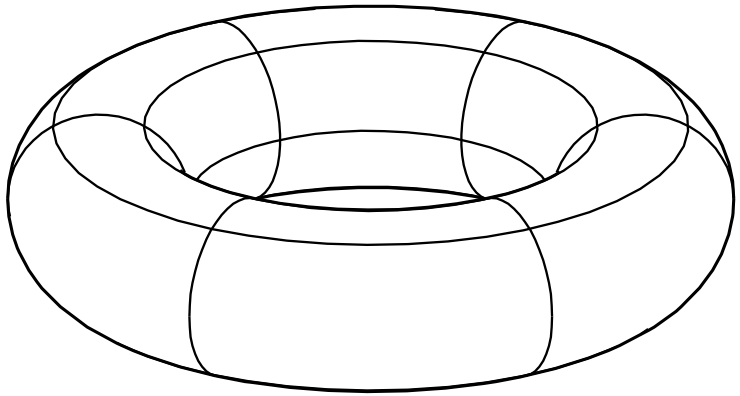
(1) Maple,Mathematica ⇒ Scilab[1],R[2]

From passing functions to passing strings

(2) Creating TEXmacros(TEXPrograming)[4][6][5]

layer Environmemnt (Free layout on the page)

```
\begin{layer}{130}{0}
\putnotese{75}{5}{
  \input{fig/figtorus_2_2}}
\end{layer}
```

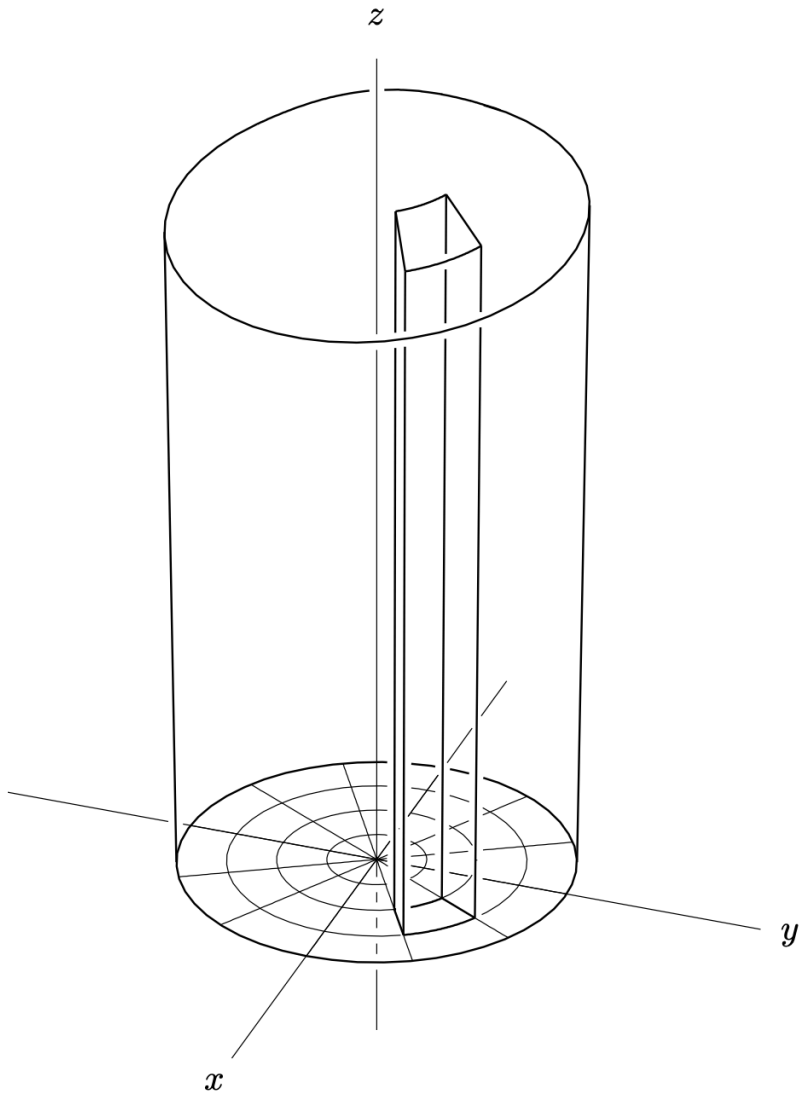


Materials Distributed in Classes

- Teachers can create beautiful figures with monochrome line drawings
  - Teachers can reduce printing costs
  - Accurate figures stimulate students' imaginations
  - Students can write comments, calculations, etc. in the empty space
- It is important to arrange text and figures freely on the page
  - TEX style files ketpic and ketlayer are essential tools
  - I distribute handouts in almost every class

Examples of Distributed Materials(Calculus)

極座標変換



Summary of KETpic

- Teachers presented various examples of use([15],[16],[17],[18],[19],[20],[21],[22])
- It takes some time to edit figures
  - (1) To modify the source code and run it
  - (2) To check the displayed figure and to export the file
  - (3) To compile the TEXdocument and to check the figures
- I began to explore ways to make a series of tasks more interactive and easier to perform

Development KETCindy

Development history of KETCindy

Computer Algebra and Dynamic Geometry in Mathematics Education

- We want to create TEXfigures easily and interactively
- In 2006, Cinderella2 was released.

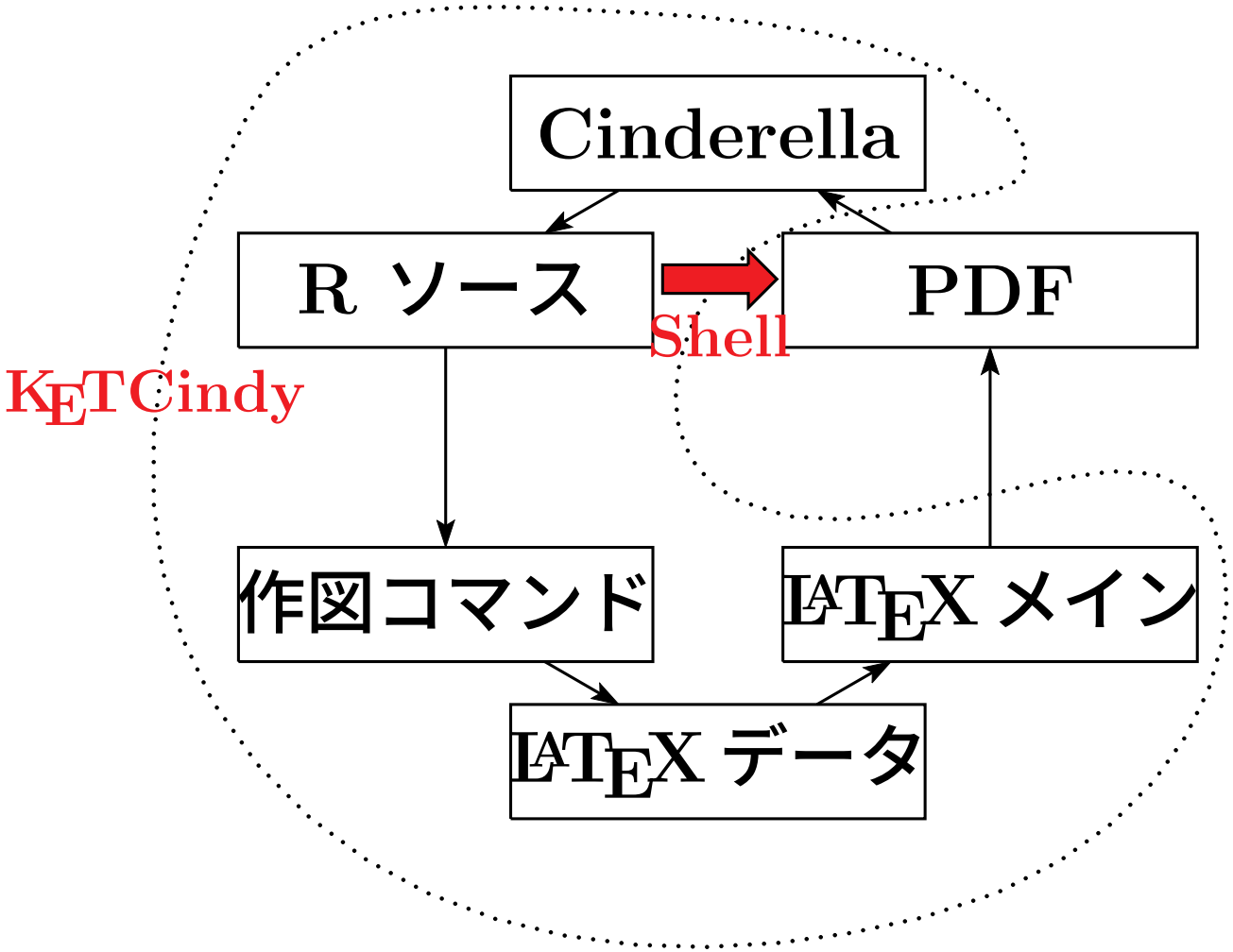
  They added the general-purpose programming language CindyScript
- I attended and presented at CADGME from 2007

  I participated in WS held by Kortenkamp(2012)

  - He is the main developer of Cinderella
- I invited him to Japan in 2014

  KETCindy was born as a result of a seminar held at Toho University

Flow Using KETCindy



KETCindy(Demo)

- https://s-takato.github.io/specialclass/shibaura25/index1.html

KETCindy Features and Enhancements

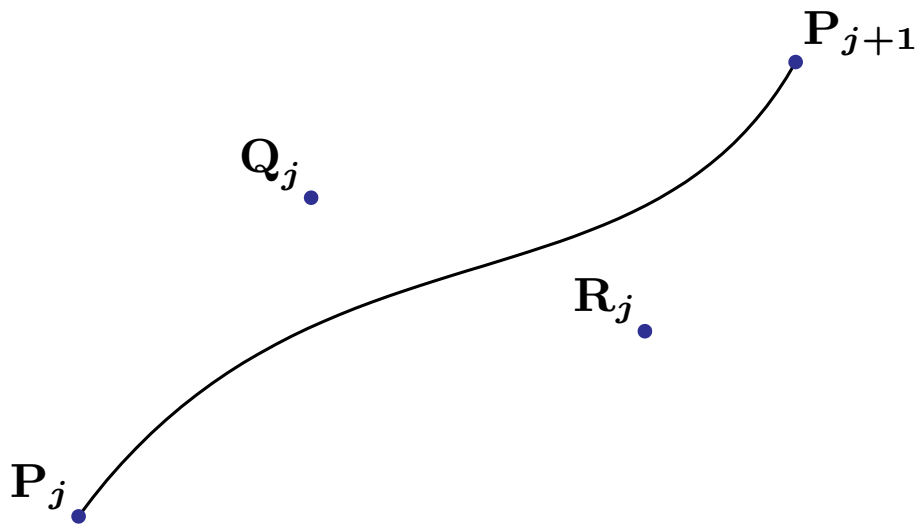
- Interactive drawing
  - Uses geometric elements of Cindy
  - Creates code files at the touch of button Figure
- loads ketcindyplugin.jar first
  - runs external programs in batches
- supports a variety of Bezier curves
- supports not only tpic but also pict2e and TikZ

  TikZ commands can also be used with あ/tt "Texcom"

Bezier Curves(cubic)

- Nodes$\mathbf{P}_j, \mathbf{P}_{j+1}$,Control Points$\mathbf{Q}_j, \mathbf{R}_j$

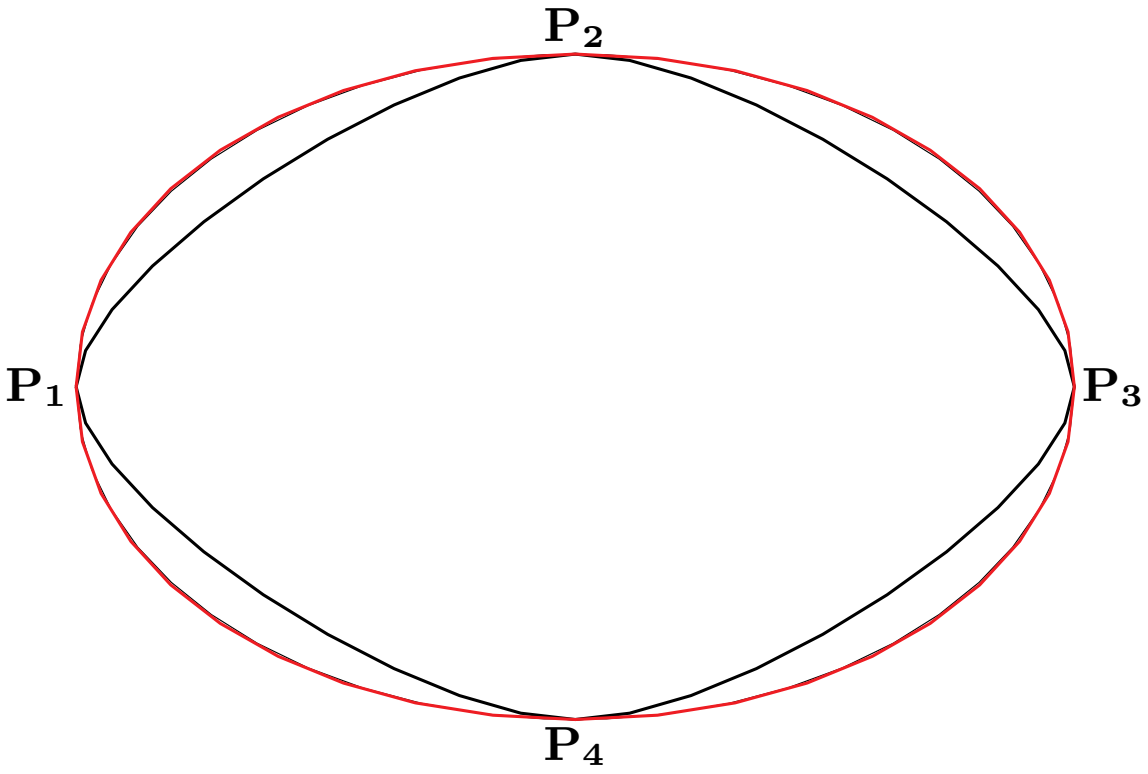
$$\mathbf{P}(t) = \mathbf{P}_j(1-t)^3 + 3\mathbf{Q}_j t(1-t)^2 + 3\mathbf{R}_j t^2(1-t) + \mathbf{P}_{j+1}t^3$$



Bezier Curves by Taking only Nodes

Approximate the following ellipse

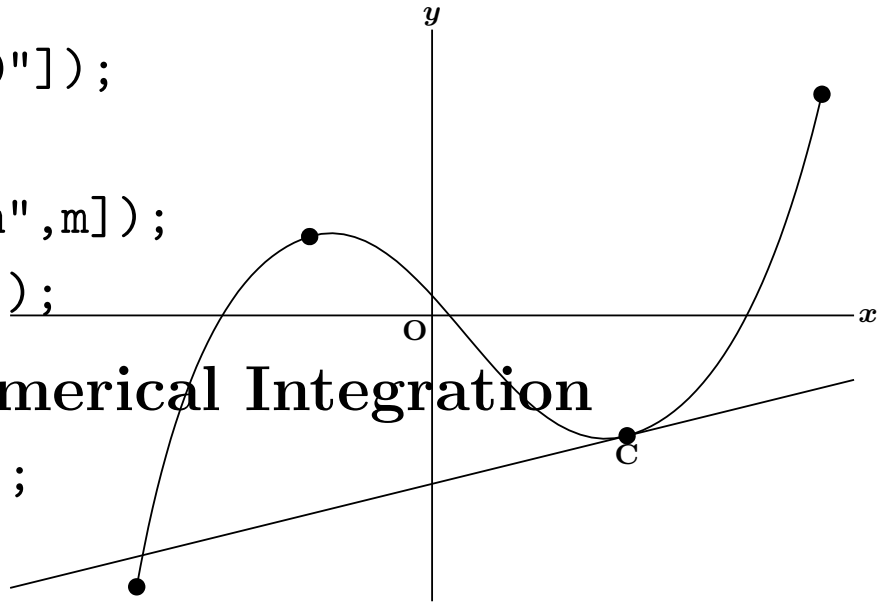
- Catmul-Rom Spline(black)
- Oshima Spline(red)
  [24],[25]



Use of Oshima Spline [29]

(1) Free Curves

```
Ospline("1",[A,B,C,D],["Num=30"]);
m=Derivative("bzo1","x=C.x");
fun=Assign("m*(x-C.x)+C.y",["m",m]);
Plotdata("2",fun,"x",["Num=1"]);
```



(2) Intersection of 2 curves,Numerical Integration

```
pt=Intersectcrvs("bzo1","gr2");
```

pt=[[-2.13,-1.76]];Only left

```
pt=Intersectcurves("bzo1","gr2");
```

pt=[-2.13,-1.76],[1.43,-0.88],[1.45,-0.88]

```
ptx=pt_1_1;
S1=Integrate("bzo1",[ptx,C.x]);
```

S2=Integrate(”gr2”,[ptx,C.x]);

S1-S2=4.15;

$$\int y dx = \int y \frac{dx}{dt} dt$$

Use of Oshima Spline(Demo)

- **Free Curve**
- **Intersection of 2 Curves**
- **Numeric Integration**
- `https://s-takato.github.io/specialclass/shibaura25/index2.html`

External Call Function from KETCindy

- Batch processing with source script as argument
- One can export the results as text and use it in KETCindy

(1) R

- Probability distribution,histograms,box plots

(2) Maxima[33]

- Math Processing,checking answers,solving Wasan

(3) gcc[34]

- Speeding up hidden line removal

Speeding up Surface Drawing with C (Demo)

- https://s-takato.github.io/specialclass/shibaura25/index3.html

Summary of KETCindy(1)

(1) KETCindyis a tool for linking KETpicand Cinderella

(2) Downloadable for free from the Github page

(3) College-level teachers create a variety of teaching materials

- "KeTCindy is a plug-in that uses Cinderella2 to create illustrations. I was a big fan of KeTpic, but I finally decided to switch to KeTCindy."(Yano, Nara)
- Hamaguchi(Nagano) creates OBJ viewer data and solid models for 3D shapes
- Noda(Toho Univ.) created teaching materials for curves and surfaces using C

Summary of KETCindy(2)

(4) For teachers who use TEXon a daily basis, KETCindyis a tool that makes it easy to create handouts and slides.

(5) However, there are many teachers who do not use TEXmuch but are eager to create effective teaching materials using their teaching experience.

(6) In addition, students can use the distributed printouts by writing on them, but this is a somewhat passive process.

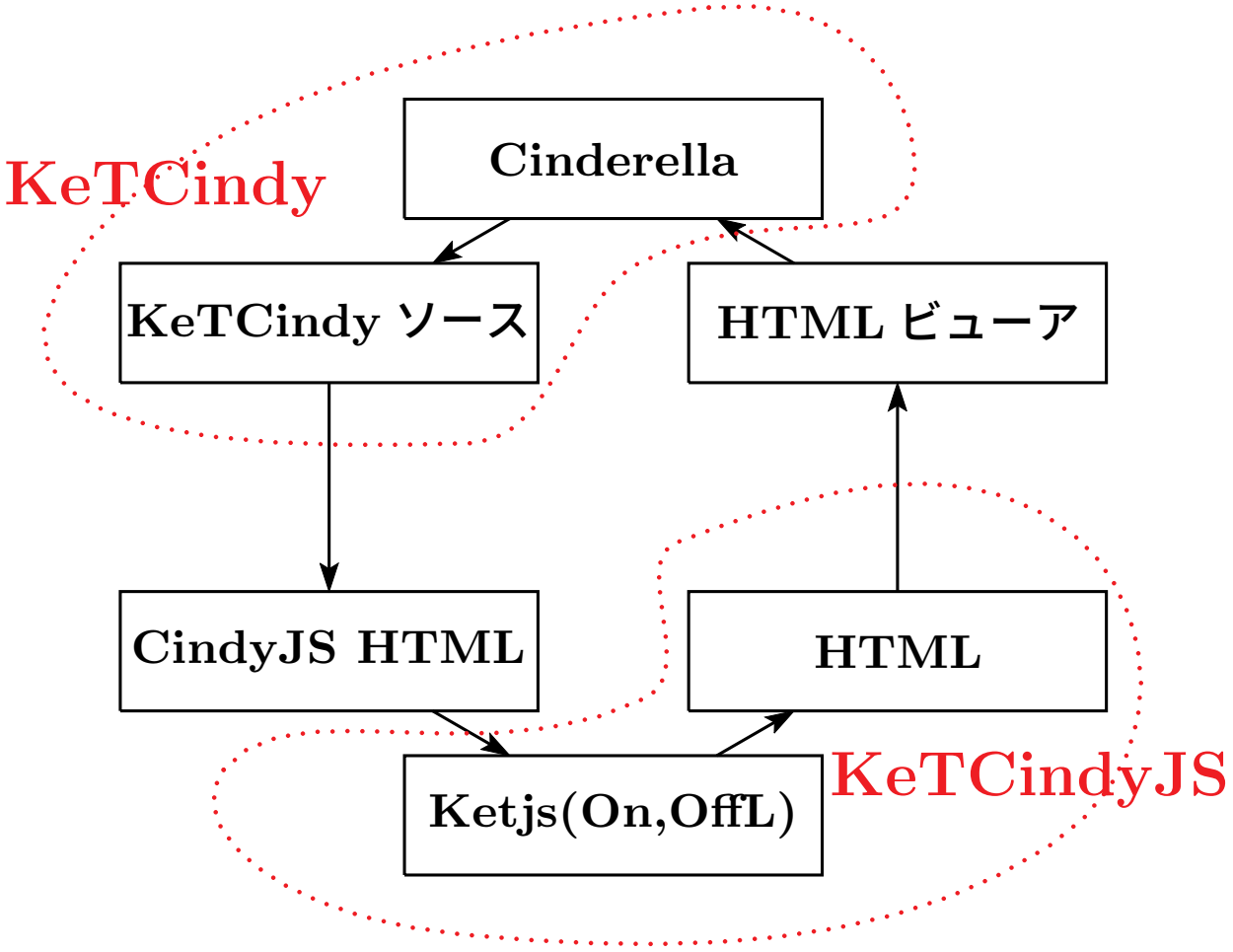
(7) We began to explore the development of apps that would allow teachers(5) and students to create teaching materials independently.

Development of KETCindyJS

Development history of KETCindyJS

- We also want teaching materials that students can use interactively
- In 2016, Cindy group released CindyJS[26]
  - It is a web framework mostly compatible with Cindy
  - It also allows for geometry and animation
  - It allows text input and output(Editable text)
- We have implemented the commands for CindyJS[27]
  - KETCindyJS first loads the CindyJS HTML file
  - Next,it looks for functions in KETCindy and embeds them recursively

Flow Using KeTCindyJS



KETCindyJS (Demo)

- https://s-takato.github.io/specialclass/shibaura25/index4.html

Examples of KETCindyJS

- No TEX compiler required
  - CindyJS supports KaTeX v0.8
- We have uploaded many samples to "ketcindy home"("ketcindy sample")
  - Brachistochrone[31]

    https://s-takato.github.io/ketcindysample/s16ketjsmisc/offline/s1611brachistchrone2jsoffL.html
  - The Basel Problem $\sum_{n=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{\pi k^2} = \frac{\pi}{6}$[38]

    https://s-takato.github.io/ketcindysample/misc/offline/basel4mainoff.html

Summary of KETCindyJS (1)

(1) The main purpose of KETCindy is to create figures for TEX documents

(2) KETCindyJS is intended to create HTML

(3) Even teachers and students who are not familiar with TEXcan create interesting teaching materials

- The KeTCindy research group of students from Numazu National College of Technology started its activities independently

  They gave a presentation on HTML teaching materials created with KETCindy(JS) at an international conference sponsored by the Nakatani Foundation, and won the top prize

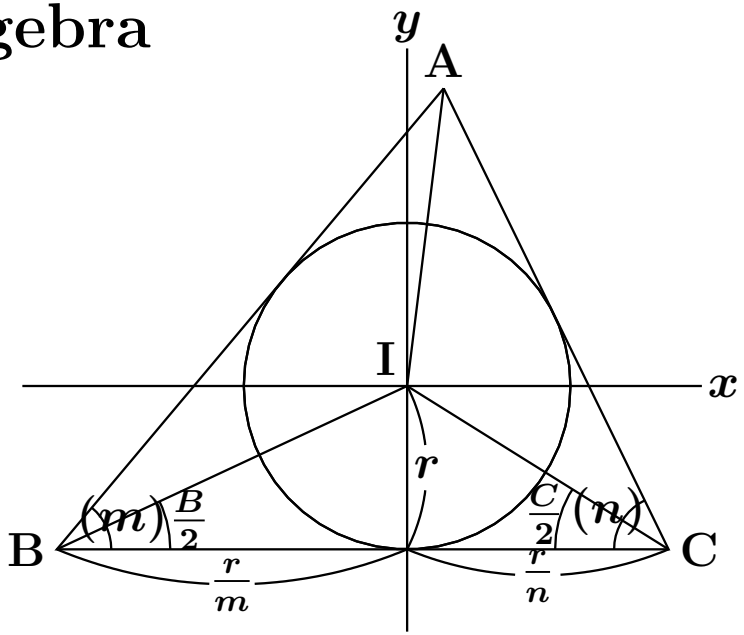
Summary of KETCindyJS (2)

(4) It is not currently possible to use symbolic computq-
ton

Solving WASAN Problems

MNR Method

- Wasan(Sangaku) problems have beautiful results but complicated calculations
- I tried to solve it by computer algebra
- Many problems involving triangles are systems of simultaneous equations that involve radicals and cannot be solved



- Therefore,I devised the MNR method[43]

Expression in the MNR method

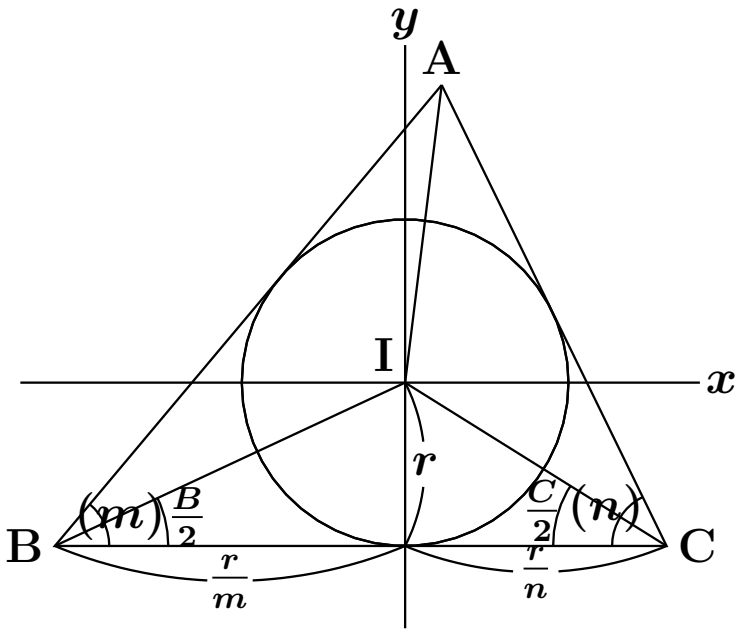
- $m = \tan\frac{B}{2}, n = \tan\frac{C}{2}$,Radius of the inscribed circle r
- The quantities of a triangle are expressed as rational expressions of m, n, r

  vtxL= B$(-\frac{r}{m},\ -r)$

  vtxR= C$(\frac{r}{n},\ -r)$

  edgB= BC $= \frac{r}{m} + \frac{r}{n}$

  edgL= AB $= \frac{r(1+m^2)}{m(1-nm)}$



- Angle operations

  Supplementary angle : supA(t)= $\tan\frac{\pi-\alpha}{2} = \frac{1}{t}$

  Sum of angles : plusA(t1,t2)= $\frac{t1+t2}{1-t1*t2}$

MNR package of Maxima

- Basic Commands

  putT(m,n,r) Put a triangle

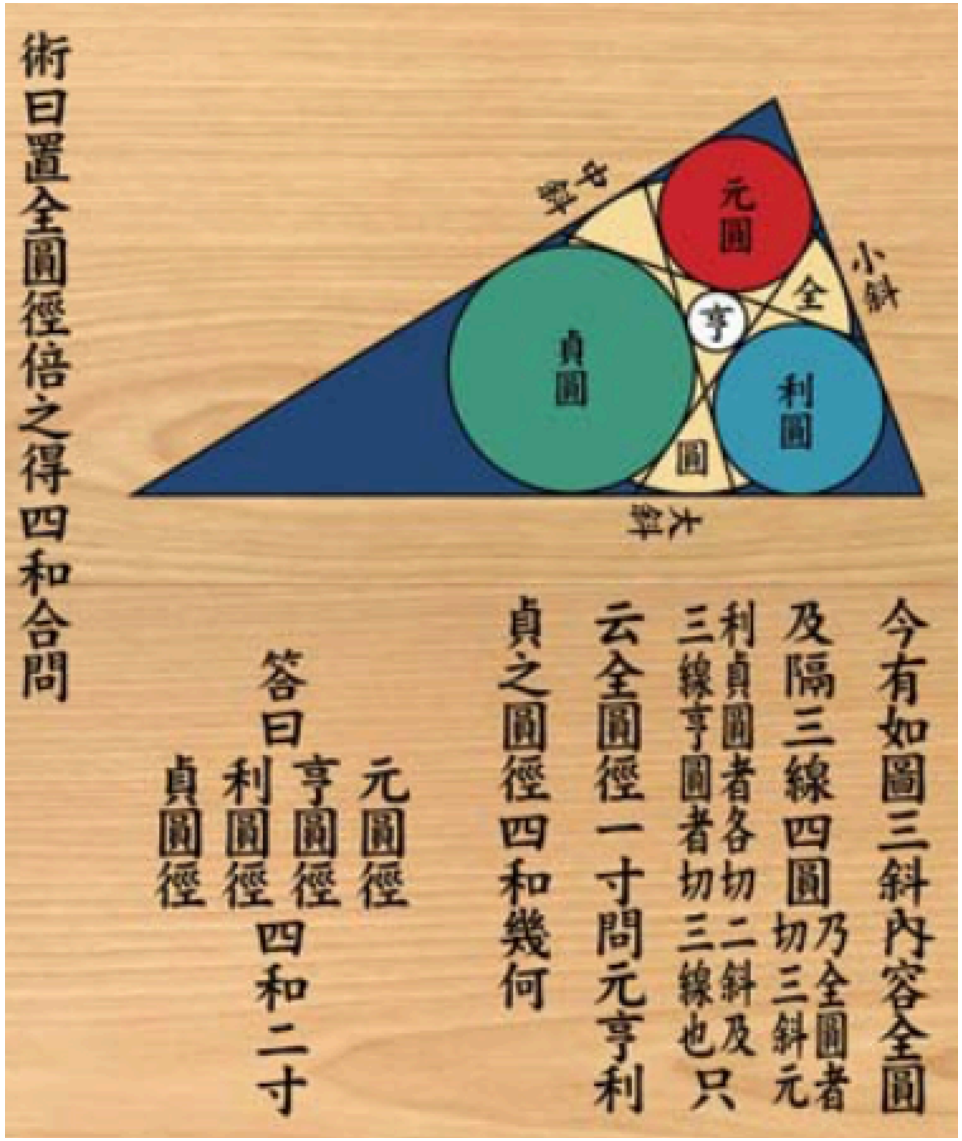
  slideT(p1,p2) Translate p1 so that it coincides with p2

  rotateT(m,p) Rotate (m) around p

  Rotation is expressed as the sine and cosine of θ, hence tan $\frac{\theta}{2}$

- Put Mxbatch("mnr") at the beginning of the Maxima command sequence [43]

Japanese Theorem (II)



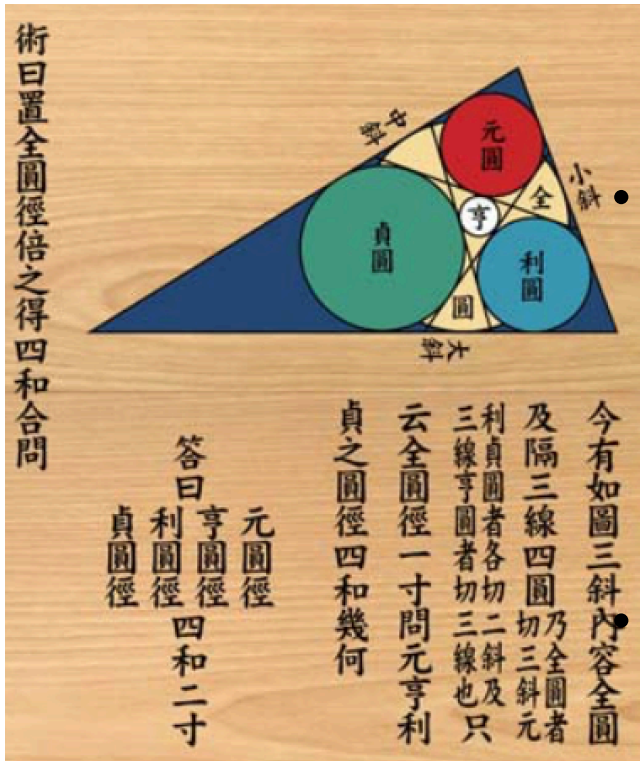
Q As shown in the figure, 全 zen circle is placed in a triangle, and four circles (元 Yuan, 利 Li, 貞 Tei, and 亨 Toh) are placed across three lines. Here, Zen is tangent to the triangle, Yuan, Li, and Tei are tangent to two sides and three lines of the triangle, and Toh is tangent to three lines. If the total diameter is 1 寸 sun, what is the sum of the diameters of Yuan, Toh, Li, and Tei?

A The sum is 2 sun

S Multiplying the total by two gives the sum

Japanese Theorem II の証明 (Demo)

- https://s-takato.github.io/specialclass/shibaura25/index5.html



Four circles (元 Yuan, 利 Li, 貞 Tei, and 亨 Toh) are placed across three lines. Here, Zen is tangent to the triangle, Yuan, Li, and Tei are tangent to two sides and three lines of the triangle, and Toh is tangent to three lines

A condition for tangent to a pentagon is required

Summary of MNR method

(1) Maxima's MNR library has been improved in interactivity with KETCindy

(2) Wasan is an essential topic when discussing the history of mathematics in Japan

- It is often mentioned in mathematics education

(3) It requires a lot of knowledge and calculation ability to actually solve it

(4) Students will be more interested and engaged if they solve the problem by themselves, even with the help of Maxima

(5) As an introduction to the MNR method, for example, the proof of the inscribed angle theorem is also possible

Conclusion

KETCindy

(1) KETCindy uses Cindy's geometry and CindyScript.
(2) Anyone can interactively create figures to insert into TEX documents (teaching materials)
(3) Batch processing runs continuously from Cindy to TEX compiler and PDF creation for confirmation
  - The Cindy and TeX files in the figure are saved separately from the main body
(4) External calls can be made (MNR,etc.)
(5) Folder structure was improved by Preining, and a subset version of TeXLive, KeTTeX, has been released by Yamamoto

KeTCindyJS

(1) KeTCindyJS is useful for creating interactive HTML teaching materials

- Even those who are unfamiliar with TeX can create a variety of teaching materials
- Web contents for textbooks:
  `https://www.dainippon-tosho.co.jp/college_math/index.html`

(2) We are developing a system called KeTLTS for sending and receiving assignment data including mathematical formulas using KeTCindyJS[43]

- Teachers and students submit their learning data using simple mathematical expressions based on TeX
- KeTLTS converts received data into 2D formulas using KaTeX and display on screen

Programming and Applications in Education

(1) Programming makes math softwares a more powerful teaching tool

(2) By creating a library, versatility can be increased

(3) If one can use selection, iteration (and function definition), one can create any diagram you want

(4) Dynamic geometry has a unique behavior (when the screen state changes, it automatically returns to the beginning and initializes variables))

(5) The application of generative AI may make programming easier, but this is a future challenge

Thank you for your attention

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