

The Solution to the Malfatti Problem in Sanpo Tenzan Tebikigusa

A Comparison between Kazuhide Omura's Method
and a Maxima-based Approach

Setsuo Takato

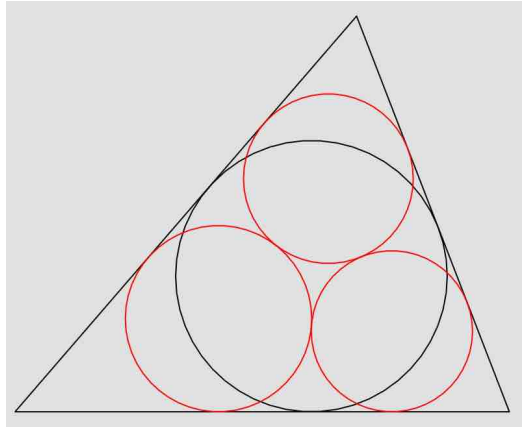
KeTCindy Center • Hon. Member of JSME

2026.09.12

Malfatti Circles

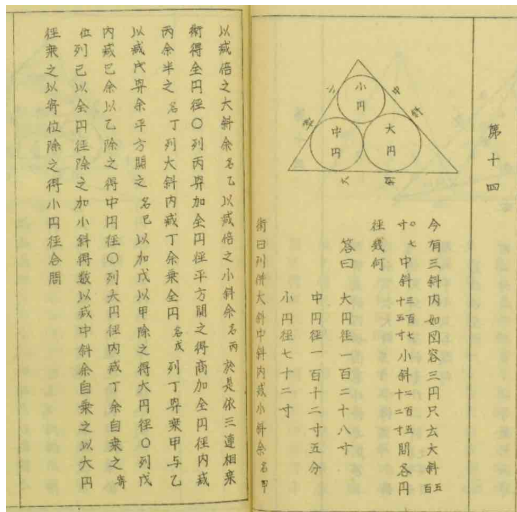
Brief History

(1) Malfatti 1803



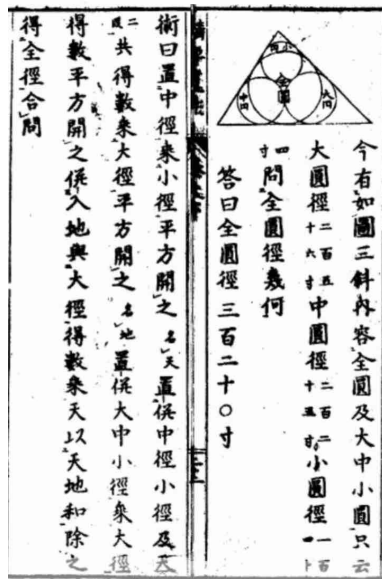
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- (1) Malfatti 1803
- (2) Ajima, Fukyu Sanpo
before 1773



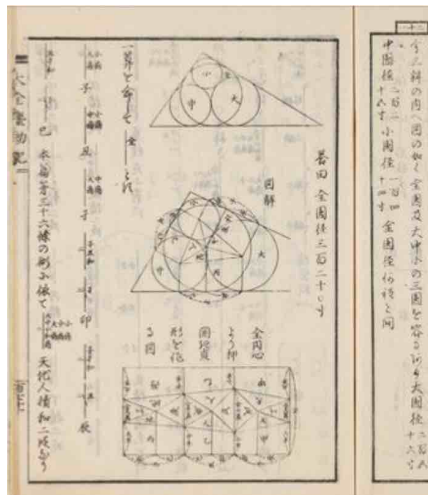
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- (4) Omura
Sanpo Tenzan Tebikigusa
1841



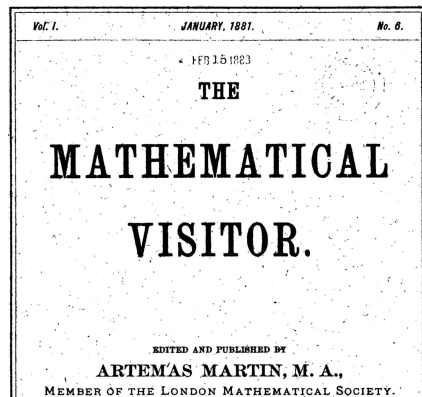
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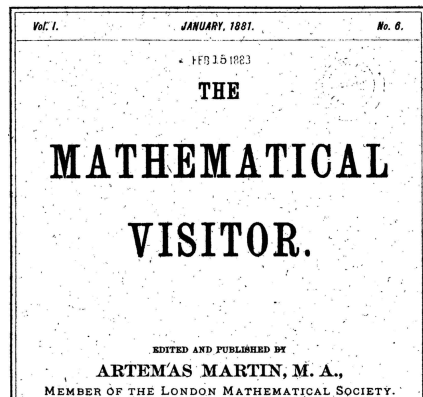
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Edmunds, Knisely, Seitz
- (7) Solution using Maxima and mnrc method 2025-



Solution in Seiyo Sanpo (Fujita)

- (1) Multiply middle and small diameters, take root, name it Ten

$$Ten = \sqrt{2r_2 \times 2r_3} = 2\sqrt{r_2 r_3}$$

- (2) Add all 3 diameters, multiply by large, take root, name it Chi

$$Chi = \sqrt{(2r_1 + 2r_2 + 2r_3) \times 2r_1} = 2\sqrt{r_1(r_1 + r_2 + r_3)}$$

- (3) Multiply large by Ten, divide by (Chi - large) to get full diameter

$$2r = \frac{2r_1 \times Ten}{Chi - 2r_1}$$

- (4) Substitute (1) and (2) into Ten and Chi of (3) and simplify

$$2r = \frac{2r_1 \times 2\sqrt{r_2 r_3}}{2\sqrt{r_1(r_1 + r_2 + r_3)} - 2r_1} = \frac{2r_1 \sqrt{r_2 r_3}}{\sqrt{r_1(r_1 + r_2 + r_3)} - r_1}$$

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Derivation in SanpoTenzanTebikigusa

D_l, D_m, D_s : Diameters of 3 circles

(1) Definition of variables

$$Ne = \sqrt{D_l \cdot D_s}, Ushi = \sqrt{D_m \cdot D_s}$$

$$Ne + Ushi = U(, Tatsu, Mi)$$

(2) Construction of relations (Equation setup)

Write direct product on left

Eliminate direct product/solve”

(3) Elimination and derivation of algorithm

(4) Multiply middle and small diameters,take root(Rin)

$$Rin = \sqrt{D_m \cdot D_s}, Ho = \sqrt{(D_l + D_m + D_s) \cdot D_l}$$

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$$D = \frac{Rin \cdot D_l}{Ho - D_l}$$

Solution by KeTCindy

What is K_ET Cindy?

- Macro package for DGS Cinderella

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- Interactively create T_EXdrawing files
 - (1) Create source code in Cinderella
 - (2) Execute T_EX via batch file by pressing a button
 - (3) Call Maxima (CAS), R, C (Surfaces), etc.

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- Distribution of K_ET Cindy
 - Search for "ketcindy home"
 - KeTTeX ($\subset$ TeXLive) is also distributed
 - Students from Numazu and Kisarazu KOSEN created a batch installer

What is KeTCindy

- Macro package for D

- Interactively create T

(1) Create source co

(2) Execute T_EX via

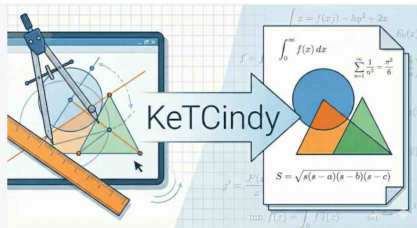
(3) Call Maxima (C

- Distribution of KeTC

• Search for "ketcin

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KeTCindy Home

KeTCindy Center

2026-08-19

最新ニュース

- 2026-08-19 KeTCindy が更新されました (version 4.5.56)

- ketslideが%ualatex対応になりました。
- KeTCindyのインストールを参考にしてください。
- 必須環境: TeX (KeTTeX, TeXLive) と R
- Windowsの場合: Sumatra PDFも必要です。

重要: インストール実行時に「すべてのユーザに対してインストール」を選択してください。(インストール先が

C:\Program Files\SumatraPDF になります)

- 2026-06-12 Cinderella が更新されました (build2136)

- Cinderellaのインストールを参考にしてください。
- 検索時は beta cinderella としてください。

- 2026-05-19 KeTTeX が TeXLive20260313 に更新されました

- 「kettex」または「kettex release」で検索可能です。
- KeTTeXのインストール / エディタ設定(TeXWorks等)

- Windows用 一括インストーラ (沼津高専 学生作成)

- Cinderella, R, Sumatra, KeTTeX, KeTCindy が一括でインストールされます。

Wasan and Solution by Maxima

- Wasan geometry are interesting teaching materials. Solving them hands-on will increase interest and engagement

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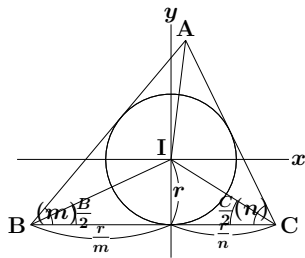
- Wasan geometry are interesting teaching materials. Solving them hands-on will increase interest and engagement
- Solving directly requires insight and considerable calculation skills
- Using CAS, high school level geometry knowledge is sufficient
- Even with CAS, it is often not easy to solve
 - Calculations involving radicals usually fail

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- Solving directly requires insight and considerable calculation skills
- Using CAS, high school level geometry knowledge is sufficient
- Even with CAS, it is often not easy to solve
 - Calculations involving radicals usually fail
- Thus, we devised the mnrc method to express triangles using $m = \tan \frac{B}{2}$, $n = \tan \frac{C}{2}$, and in-radius r .

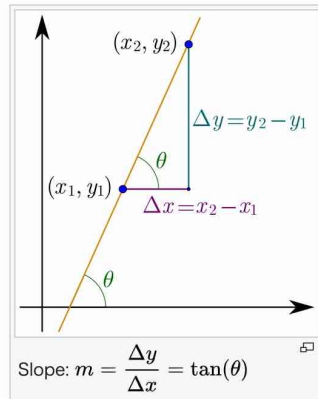
The mnR Method

- $m = \tan \frac{B}{2}, n = \tan \frac{C}{2}$
 $r = \text{inradius}$



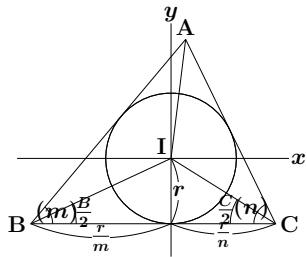
The mnr Method

- $m = \tan \frac{B}{2}, n = \tan \frac{C}{2}$
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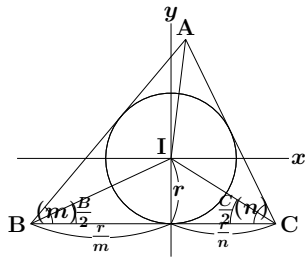
The mnR Method

- $m = \tan \frac{B}{2}, n = \tan \frac{C}{2}$
 $r = \text{inradius}$
 $\angle B = (m), \angle C = (n)$



The mn Method

- $m = \tan \frac{B}{2}, n = \tan \frac{C}{2}$
 $r = \text{inradius}$
 $\angle B = (m), \angle C = (n)$



- Expressions of triangle properties (Rational expressions)

$$\text{vtxL}:(-\frac{r}{m}, -r), \text{vtxR}:(\frac{r}{n}, -r), \text{vtxT}:(\frac{r(n-m)}{1-mn}, \frac{1+mn}{1-mn})$$

$$\text{edgB}:\frac{r}{m} + \frac{r}{n}, \text{edgL}=\frac{r(1+m^2)}{m(1-mn)}, \text{edgR}:\frac{r(1+n^2)}{n(1-mn)}$$

Incircle inC, inR, Circumcircle cirC, cirR

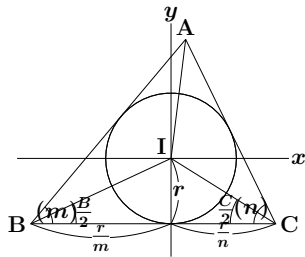
Excircles exCa, exRa, exCb, exRb, exCc, exRc

The mnR Method

- $m = \tan \frac{B}{2}, n = \tan \frac{C}{2}$

$r = \text{inradius}$

$$\angle B = (m), \angle C = (n)$$



- Expressions of triangle properties (Rational expressions)

$$\text{vtxL}:(-\frac{r}{m}, -r), \text{vtxR}:(\frac{r}{n}, -r), \text{vtxT}:(\frac{r(n-m)}{1-mn}, \frac{1+mn}{1-mn})$$

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Incircle inC, inR, Circumcircle cirC, cirR

Excircles exCa, exRa, exCb, exRb, exCc, exRc

- Created Maxima library for the mnR method

Main Commands of mnr Method

- Basic commands

putT(m,n,r) Place triangle with incenter at origin

slideT(p1,p2) Translate so that p1 matches p2

rotateT(m,p) Rotate by $2 \tan^{-1}(m)$ around p

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- Main commands for formula manipulation

numerf(f),denomf(f) Factorize num and denom

frfactor(f) Simplify fraction

frev(eq,rep) Substitute rep into eq and simplify

radcan(s) Simplify irrational expressions(Maxima)

Main Commands for Geometry

- Angle conversions

$\text{supA}(m), \text{comA}(m)$ Supple./comple. angles

$\text{plusA}(m1, m2), \text{minusA}(m1, m2)$ Sum/diff. of angles

$\text{timesA}(nn, m)$ Integer multiples of angles

- Others

$\text{dotProd}(v1, v2), \text{crossProd}(v1, v2)$ Inner/outer prod

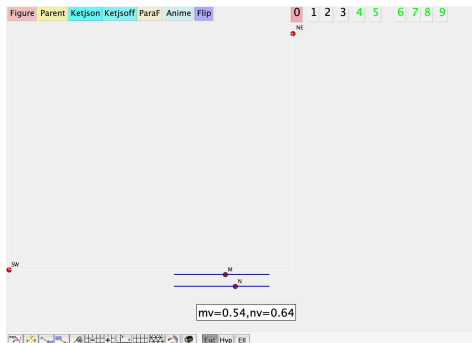
$\text{lenSeg2}(p1, p2)$ Square of distance

$\text{meetLine}(pts1, pts2)$ Intersection point

$\text{comTan1}, \text{comTan1C}$ Common outer/inner tangent

Solution by KeTCindy 1

- (1) Prepare Cinderella (cdy) file and mkcmd file
e.g., malfattiin.cdy, malfattiinmkcmd.txt
- (2) Use template file for cdy (import mkcmd.txt)



Screen of malfattiin.cdy

```

1 Ketinit();
2 Setwindow([-7,5],[-5,5]); Addax(0);
3 Pos=NE.xy+[0.5,-0.5]; Dy=1;
4 Slider("M",[0.000,-5.2],[4,-5.2]);
5 Slider("N",[0.000,-5.7],[4,-5.7]);
6 Text2.xy=[1,-7];
7 mv=M.x/4; nv=N.x/4;
8 Subsedit(2,"mv="+Sprintf(mv,2)
9           +",nv="+Sprintf(nv,2));
10 if(contains(Ch,1),Execstep1());
11 if(contains(Ch,2),Execstep2());
12 if(contains(Ch,3),Execstep3());
13 Windispg();
  
```

Ketjava 20181125
 ketcindylibbasic1[20230813] loaded
 ketcindylibbasic2[20260215] loaded

figures slot

Solution by KeTCindy 2

(3) Description in mkcmd 1

```
//step 1↓
mkcmd1()=(↓
  cmdL1=concat(Mxbatch("mnr"),[↓
    "putT(m,n,r)",↓
    "A:vtxT; B:vtxL; C:vtxR; I:inC",↓
    "AB:edgL; BC:edgB; AC:edgR",↓
    "eq1:numerf(r1+r2-AB); eq2:numerf(r2+r3-BC); eq3:numerf(r3+r1-AC)",↓
    "sol:algsys([eq1,eq2,eq3],[r1,r2,r3])",↓
    "r1:frev(r1,sol);r2:frev(r2,sol);r3:frev(r3,sol)",↓
    "make:0",↓
    "end"↓
  ]);↓
);↓
var1="A::B::C::eq1::eq2::eq3::sol::r1::r2::r3";↓
Execstep1()=(↓
  Setmnrstep(1);↓
  mkcmd1();↓
  CalcbyMset(var1,"mxans1",cmdL1,op(25));↓
  Disptex1(); Dispfig1();↓
);↓
```

Solution by KeTCindy 2

(4) Description in mkcmd 2

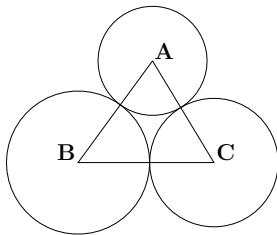
```

Disptex(Pos,Dy,"ABCをおく",opsb);↓
Disptex([Pos_1+0.5,Pos_2],Dy*0.9,"A::B::C",opsf);↓
Disptex([Pos_1-0.5,Pos_2],Dy,"3円が接する連立方程式",opsb);↓
Disptex([Pos_1+0.5,Pos_2],Dy*0.9,"eq1::eq2::eq3",opsf);↓
Disptex([Pos_1-0.5,Pos_2],Dy,"連立方程式の解",opsb);↓
Disptex([Pos_1+0.5,Pos_2],Dy*0.9,"sol",opsf);↓
Disptex([Pos_1-0.5,Pos_2],Dy,"3円の半径",opsb);↓
Disptex([Pos_1+0.5,Pos_2],Dy*0.9,"r1::r2::r3",opsf);↓
);↓
Dispfig1():=(↓
  Assignvalue();↓
  Parsevv("A::B::C");↓
  Listplot("1",[A,B,C,A],["do"]);↓
  Circledata("1a",[A,r1]);Circledata("1b",[B,r2]);↓
  Circledata("1c",[C,r3]);↓
  Letter([A,"ne","A",B,"nw","B",C,"ne","C"]);↓
);↓

```

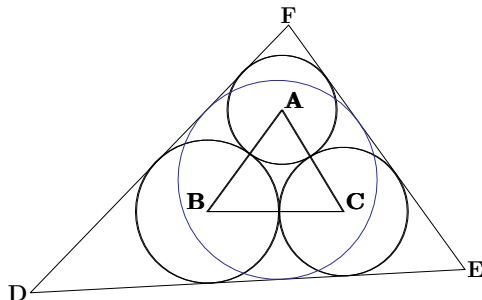
Solution from Inside (Strategy)

- (1) Place ABC using $\text{putT}(m,n,r)$
- (2) Create 3 mutually tangent circles C1,C2,C3 centered at A,B,C



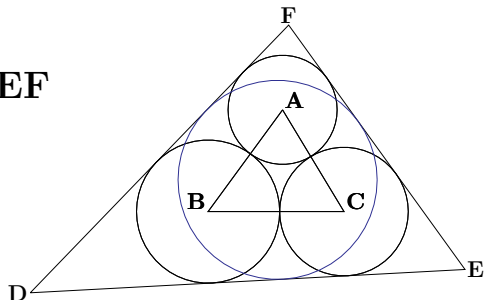
Solution from Inside (Strategy)

- (1) Place ABC using `putT(m,n,r)`
- (2) Create 3 mutually tangent circles C_1, C_2, C_3 centered at A,B,C
- (3) Create common outer tangents using `comTan`
- (4) Find intersections D,E,F of common outer tangents



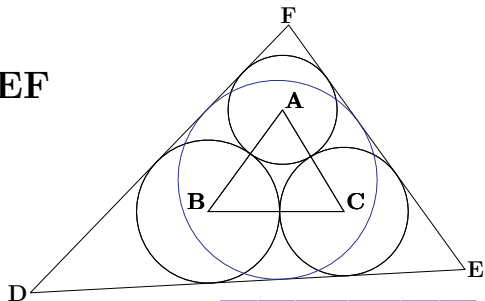
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- (5) Create incircle of DEF



Solution from Inside (Strategy)

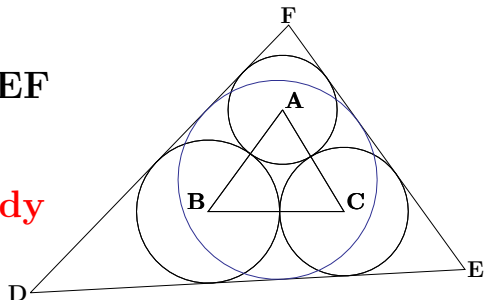
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- (6) Find the inradius



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Demo malfattiin.cdy



Solution from Inside (Result)

マウスをドラッグして自由要素を動かす

Figure Parent Ketjson Ketjsoff ParaF Anime Flip

0 1 2 3 4 5 6 7 8 9

NE

m, n, r, iR を r_1, r_2, r_3 で表す

$$\text{sol:} \left[m = \sqrt{\frac{r_1 r_3}{r_2 r_3 + r_2^2 + r_1 r_2}}, n = r_1 / ((r_3 + r_2 + r_1) \sqrt{\frac{r_1 r_3}{r_2 r_3 + r_2^2 + r_1 r_2}}), r = \frac{r_1 r_3}{(r_3 + r_2 + r_1)} \right]$$

$$m: \frac{\sqrt{r_1} \sqrt{r_3}}{\sqrt{r_2} \sqrt{r_3 + r_2 + r_1}}$$

$$n: \frac{\sqrt{r_1} \sqrt{r_2}}{\sqrt{r_3} \sqrt{r_3 + r_2 + r_1}}$$

$$r: (\sqrt{r_1} \sqrt{r_2} \sqrt{r_3}) / \sqrt{r_3 + r_2 + r_1}$$

$$iR: \sqrt{2}((2r_1 r_2^2 + 4r_1^2 r_2 + 2r_1^3) r_3^6 + (7r_1 r_2^3 + 12r_1^2 r_2^2 + 3r_1^3 r_2 - 2r_1^4) r_3^5 + (7r_1^2 r_2^3 + 12r_1^3 r_2^2 + 3r_1^4 r_2 - 2r_1^5) r_3^4 + \dots)$$

$t_1 = \sqrt{r_1}, t_2 = \sqrt{r_2}, t_3 = \sqrt{r_3}$ と置き換え

$$iRs: \frac{t_1 t_2 t_3^3 + \sqrt{t_3^2 + t_2^2 + t_1^2} (t_1 t_2 t_3^2 + t_1 t_2^2 t_3) + t_1^2 t_2 t_3^2 + (t_1 t_2^3 + t_1^2 t_2^2) t_3}{t_2 t_3 \sqrt{t_3^2 + t_2^2 + t_1^2} + t_1 t_3^2 + t_1 t_2 t_3 + t_1 t_2^2}$$

$$iRs2: \frac{t_1 t_2 t_3 (t_3^2 + t_1 t_3 + t_3 + t_2^2 + t_1 t_2 + t_2)}{t_1 t_3^2 + t_1 t_2 t_3 + t_2 t_3 + t_1 t_2^2}$$

$t = \sqrt{t_1^2 + t_2^2 + t_3^2}$ とおき分子を有理化する

$$niR: t_1 t_2 t_3^3 + t_1^2 t_2 t_3^2 + t_1 t_2 t_3^2 + t_1 t_2^3 t_3 + t_1^2 t_2^2 t_3 + t_1 t_2^2 t_3$$

$$niR2: -(2t_1^2 t_2^2 t_3^2 (t_2 t_3 - t_1 t_3 - t_1 t_2) (t_3^2 + t_2^2))$$

$$diR2: -(t_1 t_2 t_3 (t_3 + t_2 + t_1 - t) (t_2 t_3 - t_1 t_3 - t_1 t_2) (t_3^2 + t_2^2))$$

$$iR2: \frac{2t_1 t_2 t_3}{t_3^2 + t_2^2 + t_1^2 - t}$$

$$mv=0.51, nv=0.56$$

Solution from Inside (Discussion)

$$\bullet r = \frac{2\sqrt{r_1 r_2 r_3}}{\sqrt{r_1} + \sqrt{r_2} + \sqrt{r_3} - \sqrt{r_1 + r_2 + r_3}}$$

Solution from Inside (Discussion)

- $$r = \frac{2\sqrt{r_1 r_2 r_3}}{\sqrt{r_1} + \sqrt{r_2} + \sqrt{r_3} - \sqrt{r_1 + r_2 + r_3}}$$
- Different from the solution in Seiyō Sanpo (1781)

$$2r = \frac{2r_1 \times 2\sqrt{r_2 r_3}}{2\sqrt{r_1(r_1 + r_2 + r_3)} - 2r_1} = \frac{2r_1 \sqrt{r_2 r_3}}{\sqrt{r_1(r_1 + r_2 + r_3)} - r_1}$$

Solution from Inside (Discussion)

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- Sacred Mathematics (2008) also has the correct formula

Myojorinji Sangaku

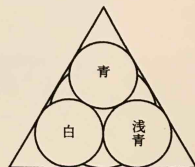
Q As shown, there are 3 circles and a Green circle (incircle) inside the triangle. Given the diameters of the 3 circles, what is the calculation method to find the diameter of the Green circle?

A Find the square root of the sum of the diameters of the 3 circles, call it 'Ten' (Heaven). Subtract 'Ten' from the sum of the square roots of the diameters of the 3 circles, call the remainder 'Jin' (Human). Multiply the diameters of the 3 circles, take the square root, multiply by 2, and divide by 'Jin' to get the diameter of the Green circle.

• Let the diameters of the 3 circles and incircle be d_1, d_2, d_3, D

$$Ten = \sqrt{d_1 + d_2 + d_3}, \quad Jin = \sqrt{d_1} + \sqrt{d_2} + \sqrt{d_3} - Ten$$

$$D = \frac{2\sqrt{d_1 d_2 d_3}}{Jin} = \frac{2\sqrt{d_1 d_2 d_3}}{\sqrt{d_1} + \sqrt{d_2} + \sqrt{d_3} - \sqrt{d_1 + d_2 + d_3}}$$

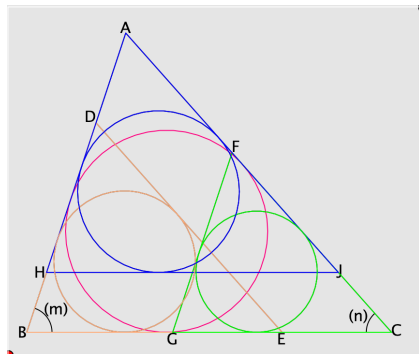


中嶋理光敬愛 志知何某貴忠 謹考
外野郷

今有如図三斜内容緑円及青・白
浅青円、只云浅青径若干白径若干
青径若干間得緑径術如何
答曰、術如左、術曰、置浅青
白青円径和開平方名天浅青白
青径各開平方得商和内減天余
名人置浅青径乘白径及青径開
平方倍之以人除之得緑径合間

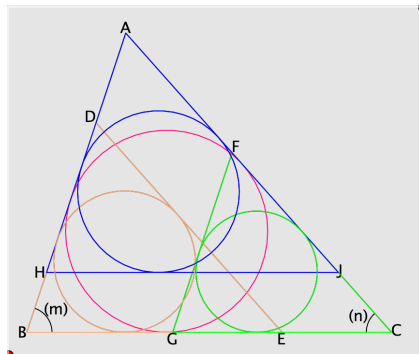
Solution from Outside (Strategy)

- (1) Place ABC using $\text{putT}(m,n,r)$, and let the in-radius be r



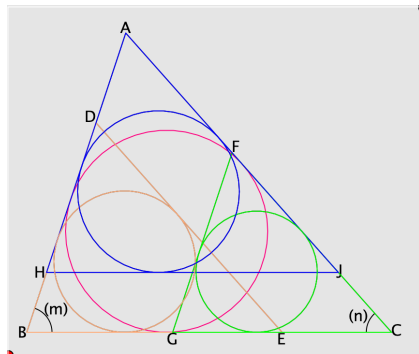
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- (1) Place ABC using $\text{putT}(m,n,r)$, and let the in-radius be r
- (2) Let r_1, r_2, r_3 be radii of incircles C_1, C_2, C_3 of AHJ, DBE, FGC similar to ABC



Solution from Outside (Strategy)

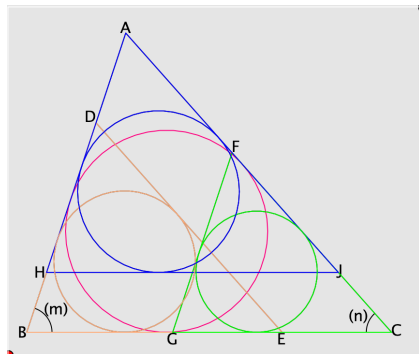
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- (2) Let r_1, r_2, r_3 be radii of incircles C_1, C_2, C_3 of AHJ, DBE, FGC similar to ABC
- (3) Solve the system of equations derived from mutually tangent conditions of C_1, C_2, C_3



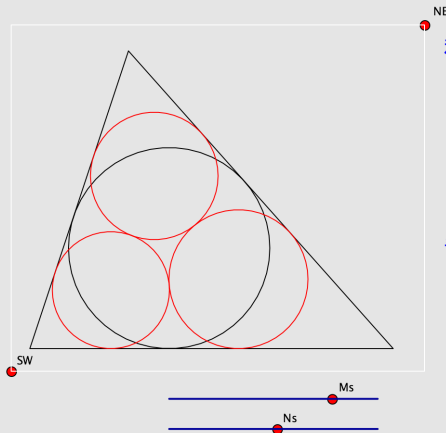
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Demo [malfattiout.cdy](#)



Solution from Outside (Result)



$$m=0.72, n=0.45$$

和算家の解答を導く

$$r1: \frac{(N+1)(MN-1)r}{(M+1)(MN-N-M-1)}$$

$$r2: \frac{(M+1)(MN-1)r}{(N+1)(MN-N-M-1)}$$

$$r3: \frac{(M+1)(N+1)(MN-N-M-1)r}{4(MN-1)}$$

置き換え $t_1 = \sqrt{s_1}, t_2 = \sqrt{s_2}, t_3 = \sqrt{s_3},$

$$\text{sol3:} [[M=(2t_1t_2t_3-r)/r, N=(2t_1t_3-r)/r]]$$

$$\text{eq5: } t_1(2t_1^2t_2^2t_3^2 - 2rt_1t_2t_3^2 - 2rt_1t_2^2t_3 - 2rt_1^2t_2t_3 + r^2t_2t_3 +$$

$$\text{sol5: } [r = - \frac{t_1t_2t_3 \sqrt{t_3^2+t_2^2+t_1^2} - t_1t_2t_3^2 + (-(t_1t_2^2) - t_1^2t_2)t_3}{(t_2+t_1)t_3+t_1t_2}]$$

$$\text{sol: } r = \frac{t_1t_2t_3 \sqrt{t_3^2+t_2^2+t_1^2} + t_1t_2t_3^2 + (t_1t_2^2 + t_1^2t_2)t_3}{(t_2+t_1)t_3+t_1t_2}$$

$$r: \frac{t_1t_2t_3(\sqrt{t_3^2+t_2^2+t_1^2} + t_3 + t_2 + t_1)}{t_2t_3 + t_1t_3 + t_1t_2}$$

Solution from Outside (Discussion)

- Matches the solution of Myojorinji

$$r: \frac{t_1 t_2 t_3 (\sqrt{t_3^2 + t_2^2 + t_1^2} + t_3 + t_2 + t_1)}{t_2 t_3 + t_1 t_3 + t_1 t_2}$$

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- Quarter angles play an important role

$$r_1: \frac{(N+1)(MN-1)r}{(M+1)(MN-N-M-1)}$$

$$r_2: \frac{(M+1)(MN-1)r}{(N+1)(MN-N-M-1)}$$

$$r_3: \frac{(M+1)(N+1)(MN-N-M-1)r}{4(MN-1)}$$

Seitz's Derivation Formula

• The Mathematical Visitor pp.190-191

III.—Solution by E. B. SEITZ, Member of the London Mathematical Society, Professor of Mathematics at the North Missouri State Normal School, Kirksville, Missouri.

Let ABC be the triangle, M, N, P the centers of the circles, D, E, F, G, H, K the points of tangency.

Let $MD = x$, $NF = y$, $PH = z$, and r = the radius of the inscribed circle of the triangle. Then $AD = AE = x \cot \frac{1}{2}A$, $BF = BG = y \cot \frac{1}{2}B$, $CH = CK = z \cot \frac{1}{2}C$,

$DF = 2\sqrt{(xy)}$, $EK = 2\sqrt{(xz)}$, $GH = 2\sqrt{(yz)}$, and we have the equations,

$$x \cot \frac{1}{2}A + 2\sqrt{(xy)} + y \cot \frac{1}{2}B = c \dots \dots \dots (1),$$

$$x \cot \frac{1}{2}A + 2\sqrt{(xz)} + z \cot \frac{1}{2}C = b \dots \dots \dots (2),$$

$$y \cot \frac{1}{2}B + 2\sqrt{(yz)} + z \cot \frac{1}{2}C = a \dots \dots \dots (3).$$

By Trigonometry we have

$$\frac{\sin \frac{1}{2}B \cos \frac{1}{2}B}{b} = \frac{\sin \frac{1}{2}C \cos \frac{1}{2}C}{c} \dots \dots \dots (4),$$

$$\frac{\sin \frac{1}{2}A \cos \frac{1}{2}A}{a} = \frac{\sin \frac{1}{2}C \cos \frac{1}{2}C}{c} \dots \dots \dots (5);$$

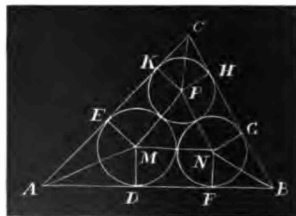
and from these two equations we can deduce the following:

$$b(\cot \frac{1}{2}A - \tan \frac{1}{2}B) = c(\cot \frac{1}{2}A - \tan \frac{1}{2}C) \dots \dots (6), \quad a(\cot \frac{1}{2}B - \tan \frac{1}{2}A) = c(\cot \frac{1}{2}B - \tan \frac{1}{2}C) \dots \dots (7).$$

Dividing (1) by (2) and (3) respectively, and clearing of fractions, we have

$$b[x \cot \frac{1}{2}A + 2\sqrt{(xy)} + y \cot \frac{1}{2}B] = c[x \cot \frac{1}{2}A + 2\sqrt{(xz)} + z \cot \frac{1}{2}C] \dots \dots \dots (8),$$

$$a[x \cot \frac{1}{2}A + 2\sqrt{(xy)} + y \cot \frac{1}{2}B] = c[y \cot \frac{1}{2}B + 2\sqrt{(yz)} + z \cot \frac{1}{2}C] \dots \dots \dots (9).$$



Seitz's Derivation Formula

Subtracting x times (6) from (8), and y times (7) from (9), we have

$$b[x \tan \frac{1}{2}B + 2\sqrt{xy} + y \cot \frac{1}{2}B] = c[x \tan \frac{1}{2}C + 2\sqrt{zx} + z \cot \frac{1}{2}C] \dots\dots\dots (10),$$

$$a[y \cot \frac{1}{2}A + 2\sqrt{xy} + y \tan \frac{1}{2}A] = c[y \tan \frac{1}{2}C + 2\sqrt{yz} + z \cot \frac{1}{2}C] \dots\dots\dots (11).$$

Multiplying (10) by (4), and (11) by (5), and extracting the square root, we have

$$\sqrt{x} \sin \frac{1}{2}B + \sqrt{y} \cos \frac{1}{2}B = \sqrt{x} \sin \frac{1}{2}C + \sqrt{z} \cos \frac{1}{2}C \dots\dots\dots (12),$$

$$\sqrt{x} \cos \frac{1}{2}A + \sqrt{y} \sin \frac{1}{2}A = \sqrt{y} \sin \frac{1}{2}C + \sqrt{z} \cos \frac{1}{2}C \dots\dots\dots (13).$$

Subtracting (13) from (12), we find

$$\frac{\sqrt{y}}{\sqrt{x}} = \frac{\sin \frac{1}{2}C + \cos \frac{1}{2}A - \sin \frac{1}{2}B}{\sin \frac{1}{2}C - \sin \frac{1}{2}A + \cos \frac{1}{2}B} = \frac{\cos \frac{1}{2}C + \cos \frac{1}{2}(2B+C)}{\cos \frac{1}{2}C + \cos \frac{1}{2}(2A+C)} = \frac{\cos \frac{1}{2}B \cos \frac{1}{2}(\pi-A)}{\cos \frac{1}{2}A \cos \frac{1}{2}(\pi-B)} = \frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}B} \dots\dots (14).$$

Similarly we find

$$\frac{\sqrt{z}}{\sqrt{x}} = \frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}C} \dots\dots\dots (15).$$

Substituting the value of $\sqrt{y}$ from (14) in (1), we have

$$\left[\cot \frac{1}{2}A + 2 \left(\frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}B} \right) + \cot \frac{1}{2}B \left(\frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}B} \right)^2 \right] x = c,$$

or
$$\left[\frac{1 - \tan^2 \frac{1}{2}A}{2 \tan \frac{1}{2}A} + 2 \left(\frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}B} \right) + \left(\frac{1 - \tan^2 \frac{1}{2}B}{2 \tan \frac{1}{2}B} \right) \left(\frac{1 + \tan \frac{1}{2}A}{1 + \tan \frac{1}{2}B} \right)^2 \right] x = c,$$

whence
$$\begin{aligned} x &= \frac{2c \tan \frac{1}{2}A \tan \frac{1}{2}B (1 + \tan \frac{1}{2}B)}{(1 + \tan \frac{1}{2}A)(\tan \frac{1}{2}A + \tan \frac{1}{2}B)(1 - \tan \frac{1}{2}A \tan \frac{1}{2}B)[1 + \tan \frac{1}{2}(A+B)]} \\ &= \frac{c \sin \frac{1}{2}A \sin \frac{1}{2}B (1 + \tan \frac{1}{2}B)}{\sin \frac{1}{2}(A+B)(1 + \tan \frac{1}{2}A)[1 + \tan \frac{1}{2}(\pi-C)]} = \frac{r(1 + \tan \frac{1}{2}B)}{(1 + \tan \frac{1}{2}A)[1 + \tan \frac{1}{2}(\pi-C)]} \\ &= \frac{\frac{1}{2}r(1 + \tan \frac{1}{2}B)(1 + \tan \frac{1}{2}C)}{1 + \tan \frac{1}{2}A}. \end{aligned}$$

Substituting the value of x in (14) and (15), we find

$$y = \frac{\frac{1}{2}r(1 + \tan \frac{1}{2}A)(1 + \tan \frac{1}{2}C)}{1 + \tan \frac{1}{2}B}, \quad \text{and} \quad z = \frac{\frac{1}{2}r(1 + \tan \frac{1}{2}A)(1 + \tan \frac{1}{2}B)}{1 + \tan \frac{1}{2}C}.$$

An excellent solution was furnished by *William Hoover*, and a very brief one by Professor *Casey*. We hope to publish other solutions in a future No.

This celebrated problem has a history, and is known to mathematicians as "Malfatti's Problem."

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$$\begin{aligned} x &= \frac{2c \tan \frac{1}{2}A \tan \frac{1}{2}B (1 + \tan \frac{1}{2}B)}{(1 + \tan \frac{1}{2}A)(\tan \frac{1}{2}A + \tan \frac{1}{2}B)(1 - \tan \frac{1}{2}A \tan \frac{1}{2}B)[1 + \tan \frac{1}{2}(A+B)]'} \\ &= \frac{c \sin \frac{1}{2}A \sin \frac{1}{2}B (1 + \tan \frac{1}{2}B)}{\sin \frac{1}{2}(A+B)(1 + \tan \frac{1}{2}A)[1 + \tan \frac{1}{2}(\pi-C)]'} = \frac{r(1 + \tan \frac{1}{2}B)}{(1 + \tan \frac{1}{2}A)[1 + \tan \frac{1}{2}(\pi-C)]'} \\ &= \frac{\frac{1}{2}r(1 + \tan \frac{1}{2}B)(1 + \tan \frac{1}{2}C)}{1 + \tan \frac{1}{2}A}. \end{aligned}$$

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Equivalence with Seitz's Formula

- Let $L = \tan(A/4)$

$$r1: \frac{(N+1)(MN-1)r}{(M+1)(MN-N-M-1)}$$

$$L = \tan(\pi/4 - B/4 - C/4) = \frac{1 - \tan(B/4 + C/4)}{1 + \tan(B/4 + C/4)}$$

$$= \frac{1 - \frac{M+N}{1-MN}}{1 + \frac{M+N}{1-MN}} = \frac{1-MN-M-N}{1-MN+M+N}$$

$$1 + L = 1 + \frac{1-MN-M-N}{1-MN+M+N} = 2 \frac{1-MN}{1-MN+M+N}$$

$$\frac{1+L}{2} = \frac{1-MN}{1-MN+M+N}$$

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$$s1 = \frac{r1}{r} = \frac{(N+1)(MN-1)}{(M+1)(MN-N-M-1)} = \frac{(1+N)(1+L)}{2(1+M)}$$

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- Therefore

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$$y = \frac{\frac{1}{2}r(1+\tan \frac{1}{4}A)(1+\tan \frac{1}{4}C)}{1+\tan \frac{1}{4}B}$$

Conclusion

- Potential as an Objective Verification Tool in Mathematics History Research
 - The mnrc method and CAS can objectively prove historical errors caused by human manual calculations (Tenzan)
 - Corrected the error in SeiyoSanpo and validated the truth of Myojorinji

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 - The mnrc method and CAS can objectively prove historical errors caused by human manual calculations (Tenzan)
 - Corrected the error in SeiyoSanpo and validated the truth of Myojorinji
- **New Approach to Wasan Education**
 - Even without advanced geometrical insight, setting up equations correctly leads to the solution. Students' interest in Wasan will further increase by "verifying" history themselves

Conclusion

- Future Work

- Expansion and refinement of the mn� method to handle more complex Sangaku problems (e.g., quarter angles)
- Development of manuals and error-checking functions to ensure beginners can reliably perform interactive solving